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ЗАМЕЧАНИЕ О $\mathfrak{X}$-ЛОКАЛЬНЫХ ФОРМАЦИЯХ

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A NOTE ON $\mathfrak{X}$-LOCAL FORMATIONS

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Установлено, что всякая $\mathfrak{X}$-локальная (в смысле Фёрстера) формация конечных групп является ω-композиционной формацией, где $\omega = \pi(\mathfrak{X})$.

Ключевые слова: *конечная группа, формация.*

It is proved that every $\mathfrak{X}$-local (by Förster) formation of finite groups is an ω-composition formation, where $\omega = \pi(\mathfrak{X})$.

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All groups considered are finite. We use standard notations (see [1]). The characteristic char($\mathfrak{X}$) of a group class $\mathfrak{X}$ is the set of primes p such that $\mathfrak{X}$ contains a group of order p; $\pi(\mathfrak{H})$ is the set of all prime divisors of groups in $\mathfrak{H}$. A chief factor H / K is called a chief $\mathfrak{H}$-factor if $H / K \in \mathfrak{H}$. We denote by $\mathrm{E}\mathfrak{H}$ the class of groups all composition factors of which belong to $\mathfrak{H}$. We denote by $\mathfrak{J}$ the class of all simple (abelian and non-abelian) groups. If $\mathfrak{X} \subseteq \mathfrak{J}$, then $\mathfrak{X}' = \mathfrak{J} \setminus \mathfrak{X}$, and $\mathfrak{X}^+$ is the class of abelian groups in $\mathfrak{X}$. We denote by $\mathcal{K}(G)$ the class of simple groups isomorphic to composition factors of a group G. By $C^A(G)$ we denote the intersection of all centralizers of all chief factors H/K of the group G such that $A \in \mathcal{K}(H/K)$ ($C^A(G) = G$ if G does not contain chief factors of this type).

The concept of $\mathfrak{X}$-local formation was introduced by P. Förster (see [2], [1, p. 374], [3, Definition 3.1.1], [4, Definition 1.3]).

***Definition* 1.** *Let* $\mathfrak{X}$ *be a class of simple groups such that* char($\mathfrak{X}$) = $\pi(\mathfrak{X})$. *Consider a function*

$$f\colon \pi(\mathfrak{X}) \cup \mathfrak{X}' \to \{\text{formations}\},$$

which we call an ***$\mathfrak{X}$-formation function***; *we assume that f takes equal values at isomorphic groups. Let* $LF_{\mathfrak{X}}(f)$ *be the class of groups G satisfying the following conditions:*

1) *if* H / K *is a chief* $\mathrm{E}\mathfrak{X}$*-factor of* G, *then* $G/C_G(H / K)$ *belongs to* $f(p)$ *for each* p in $\pi(H / K)$;

2) *if* G / L *is monolithic and* $\mathrm{Soc}(G / L) \in \mathrm{E}\mathfrak{X}'$, *then* $G / L \in f(E)$, *where* $E \in \mathcal{K}(\mathrm{Soc}(G / L))$. *The class* $LF_{\mathfrak{X}}(f)$ *is a formation*; *it is called an* ***$\mathfrak{X}$-local formation***.

***Lemma* 1** (see [4, Lemma 2.1, Lemma 4.4]). *Let* $\mathfrak{X}$ *be a class of simple groups such that* char($\mathfrak{X}$) = =$\pi(\mathfrak{X})$. Set $\mathfrak{L} = \mathfrak{X}^+$. *Let f be an* $\mathfrak{X}$*-formation function and* $\mathfrak{F} = LF_{\mathfrak{X}}(f)$. *Then* $\mathfrak{F} = LF_{\mathfrak{L}}(h)$, *where h is an* $\mathfrak{L}$*-formation function such that* $h(p) = f(p) \cap \mathfrak{F}$ *for every p in* $\pi(\mathfrak{X})$, *and* $h(E) = \mathfrak{F}$ *for every E in* $\mathfrak{L}'$.

***Lemma* 2** (see [4, Lemma 3.1]). *Let* $\mathfrak{X}$ *be a class of simple groups such that* char($\mathfrak{X}$) = $\pi(\mathfrak{X})$. *Let f be an* $\mathfrak{X}$*-formation function and* $\mathfrak{F} = LF_{\mathfrak{X}}(f)$. *Let M be a minimal normal subgroup of a group G such that* $G / M \in \mathfrak{F}$, $M \in \mathrm{E}\mathfrak{X}$ *and M is f-central in G*, i.e., $G / C_G(M) \in f(p)$ *for each p in* $\pi(M)$. *Then* $G \in \mathfrak{F}$.

The concept of $\mathfrak{L}$-composition formation was proposed in [5].

Definition* 2.** *Let* $\mathfrak{L}$ *be an arbitrary non-empty class of simple groups. Then any function f of the form* $f\colon \mathfrak{L} \cup \{\mathfrak{L}'\} \to \{\text{formations}\}$ *taking equal values on the isomorphic groups is called an* ***$\mathfrak{L}$-composition satellite. *If A is an* $\mathfrak{L}$*-group of prime order p, we write* $f(p)$ *instead of* $f(A)$. *For any* $\mathfrak{L}$*-composition satellite f, we denote by* $CF_{\mathfrak{L}}(f)$ *the class of groups G satisfying the following conditions:*

1) $G / G_{\mathrm{E}\mathfrak{L}} \in f(\mathfrak{L}')$, *where* $G_{\mathrm{E}\mathfrak{L}}$ *is the* $\mathrm{E}\mathfrak{L}$*-radical of G*;

2) $G / C^A(G) \in f(A)$ *for every* $A \in \mathcal{K}(G) \cap \mathfrak{L}$.

The class $CF_{\mathfrak{L}}(f)$ *is a formation; it is called an* ***$\mathfrak{L}$-composition formation***. *If* $\mathfrak{L} = \mathfrak{L}^+$ *and* $\omega = \pi(\mathfrak{L})$, *the class* $CF_{\mathfrak{L}}(f)$ *is called an* ***ω-composition formation***.

Theorem. *Let* $\mathfrak{X}$ *be a non-empty class of simple groups such that* $\omega = \mathrm{char}(\mathfrak{X}) = \pi(\mathfrak{X})$. *Then every* $\mathfrak{X}$*-local formation is an* ω*-composition formation.*

Proof. Let $\mathfrak{F} = LF_{\mathfrak{X}}(f)$. Set $\mathfrak{L} = \mathfrak{X}^{+}$. By Lemma 1, $\mathfrak{F} = LF_{\mathfrak{L}}(h)$, where h is an $\mathfrak{L}$-formation function such that $h(p) = f(p) \cap \mathfrak{F}$ for every p in $\pi(\mathfrak{L}) = \omega$, and $h(E) = \mathfrak{F}$ for every E in $\mathfrak{L}'$. Now we consider an $\mathfrak{L}$-composition formation $\mathfrak{D} = CF_{\mathfrak{L}}(d)$, where d is an $\mathfrak{L}$-composition satellite such that $d(p) = h(p)$ for every p in ω, and $d(\mathfrak{L}') = \mathfrak{F}$. We prove that $\mathfrak{F} = \mathfrak{D}$.

If $G \in \mathfrak{F}$, then $G / G_{E\mathfrak{L}} \in \mathfrak{F} = d(\mathfrak{L}')$ and for every chief $(\mathcal{K}(G) \cap \mathfrak{L})$-factor H / K of G we have that $G/C_G(H / K)$ belongs to $d(p) = h(p)$ where p in $\pi(H / K)$. Thus, G belongs to $\mathfrak{D}$. So, $\mathfrak{F} \subseteq \mathfrak{D}$.

Let G be the group of the least order in $\mathfrak{D} \setminus \mathfrak{F}$. Then $L = G^{\mathfrak{F}}$ is the socle of G. If $G_{E\mathfrak{L}} \neq 1$, then L belongs to $E\mathfrak{L}$, and by Lemma 2 we have $G \in \mathfrak{F}$. Assume that $G_{E\mathfrak{L}} = 1$. Then, according to Definition 2, G belongs to $f(\mathfrak{L}') = \mathfrak{F}$.

The theorem is proved.

Remark. In [5] it was proved that every $\mathfrak{L}$-composition formation $\mathfrak{F}$ is a p-composition formation for any *p in* $\pi(\mathfrak{L}^{+})$, so $\mathfrak{F}$ is $\mathfrak{L}^{+}$-local (see also [3, p. 152]).

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